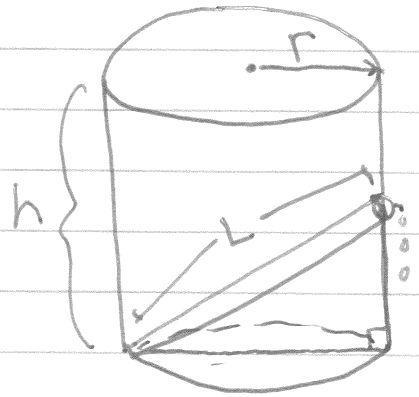
Class 16: Constrained Optimization

Reminder: Resources for OSH/WeBWork:
- Office Hours (Mine or others)
- MLC (next to LSK 300)
- Email me Questions
  Ex. "I found Problem X hard"
- Piazza.

Today:
- Practice constrained optimization (#1, 2, 3)
  Given that $\square = 15$, maximize Δ.
- If time #4 unconstrained optimization
  #5 Sketching

Constraint

objective function



Constraint: $L = \sqrt{(2r)^2 + \left(\frac{h}{2}\right)^2}$

Objective function: Volume $= \pi r^2 h$

Variables: r, h

- Isolate one variable in Constraint equation

$$L^2 = 4r^2 + \frac{h^2}{4} \Longrightarrow L^2 - \frac{h^2}{4} = 4r^2$$

$$\Longrightarrow r^2 = \frac{L^2}{4} - \frac{h^2}{16}$$

$$\Longrightarrow \boxed{r = \pm\sqrt{\frac{L^2}{4} - \frac{h^2}{16}}}$$

- Plug into objective function.

$$f(h) = \text{Volume} = \pi\left(\frac{L^2}{4} - \frac{h^2}{16}\right)h = \boxed{\frac{\pi L^2}{4}h - \frac{\pi}{16}h^3}$$

- Find the global maximum of $f(h)$

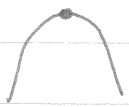
$$f'(h) = \frac{\pi L^2}{4} - \frac{3\pi}{16}h^2 = 0 \Rightarrow \text{Critical point}$$

h is in $(0, 2L)$

$$\boxed{h = \frac{2}{\sqrt{3}}L}$$

Is $h = \frac{2}{\sqrt{3}}L$ a max, or a min? Can test two ways

- $f''(h) = -\frac{3\pi}{8}h$ is negative at $h = \frac{2}{\sqrt{3}}L$



local max

- OR plug in values. $0 < h < 2L$

$h = 0$: $f(0) = 0$

$h = \frac{2}{\sqrt{3}}L$: $f\left(\frac{2}{\sqrt{3}}L\right) = \frac{\pi}{2\sqrt{3}}L^3 - \frac{\pi}{6\sqrt{3}}L^3 = \frac{\pi}{3\sqrt{3}}L^3$

$h = 2L$: $f(2L) = \frac{2\pi L^3}{2} - \frac{\pi L^3}{2} = 0$